Sequence

$f: N \to X$ — Sequence if $X = R$ real Sequence
if $X = C$ Complex Sequence.

$a: N \to R$ be Sequence. elements are $a_1, a_2, a_3 \ldots$ all distinct.

$\{a_n\}$ $n \in N$, or $\{a_n\}$ or (a_n) or $\langle a_n \rangle$

Range of seq: $R = \{a_n, n \in N\}$ without repetition and irrespective of their position.

Eg ① Seq. $\{a_n\}$ when $a_n = n$ is $\{1, 2, 3, \ldots, n, \ldots\}$

② $a_n = \frac{1}{n}$ — $\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\}$

③ $a_n = (-1)^n = \{-1, 1, -1, 1, \ldots\}$ Range $= \{1, -1\}$

Note - ① a_n nth and mth for $m \neq n$ are distinct even if $a_n = a_m$

② seq. always infinite.

③ if zeroth term added then Domain $N \cup \{0\}$

Equal seq.

$\{a_n\}$ and $\{b_n\}$ seq^ns are said to be equal if $a_n = b_n$ for every n.

Bounded and unbounded Seq^n.

1) $\{a_n\}$ bounded above if $\exists$ real no. k $\ni$

$a_n \leq k \quad \forall \quad n \in N$

k = uper bound.

) $\{a_n\}$ bounded below if $\exists$ h real No. $\ni$

$h \leq a_n \;\forall\; n \in N$. h = lower bound.

3) $\{a_n\}$ Bounded. if $\exists$, h & k real No. $\ni$

$h \leq a_n \leq k \quad n \in N$

4) if Not bounded then Unbounded.

Note:-> ① $\{a_n\}$ - Bounded, unbounded $\longrightarrow$ Range - Bounded, unbou

Convergent Sequence.

Seq. $\{a_n\}$ is Said to converge to a Limit l if $a_n \longrightarrow l$ as $n \longrightarrow \infty$

i.e l is limit of the seq^n.

$\lim\limits_{n\to\infty} a_n = l$ or $\lim a_n = l$

$a_n \longrightarrow l$ as $n \longrightarrow \infty$ or $a_n \longrightarrow l$

Eg. ① $\{a_n\} = \left\{\frac{1}{n}\right\}$ ② $a_n = \frac{n}{n+1}$

$n \to \infty \quad a_n \to 0$ $\qquad n \to \infty \quad a_n \to 1$

Note. Seq $\{a_n\}$ Convergent if $\lim\limits_{n\to\infty} a_n$ is finite.

Divergent Sequence:

if $\lim a_n \longrightarrow \infty$

or $\longrightarrow -\infty$

Eg ① $a_n = 2n^2 + n$ ② $a_n = n$ ③ $a_n = \frac{n^2}{n+1}$

Oscillatory Seq^n.

neither Convergent nor divergent.

1) bounded $\{a_n\}$ — oscillate finite eg $a_n = (-1)^n$

2) Unbounded $\{a_n\}$ — Oscillate Infinitely eg $a_n = n(-1)^n$

Algebra of Limits. ($a_n \to a$ $b_n \to b$ then)

① $\lim_{n\to\infty} |a_n| = |a|$ ② $\lim_{n\to\infty} (-a_n) = -a$

③ $\lim_{n\to\infty} (ka_n) = ka$ ④ $\lim_{n\to\infty} (a_n + b_n) = a+b$

⑤ $\lim_{n\to\infty} (a_n - b_n) = a-b$ ⑥ $\lim_{n\to\infty} (a_n b_n) = ab$

⑦ $\lim_{n\to\infty} \left(\frac{1}{b_n}\right) = \frac{1}{b}$ $b_n \neq 0$ $b \neq 0$ ⑧ $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \frac{a}{b}$

$b_n \neq 0$ $b \neq 0$

★ Monotonic Sequences.

1) A sequence $\{a_n\}$ is said to be monotonically increasing if $a_n \leq a_{n+1}$ $\forall$ $n \in N$.

ii) A sequence $\{a_n\}$ is said to be monotonically decreasing if $a_n \geq a_{n+1}$ $\forall$ $n \in N$.

(iii) A sequence which is monotonically increasing or decreasing is called a monotonic (or a monotone) sequence.

(iv) A sequence $\{a_n\}$ is said to be strictly monotonically increasing if $a_n < a_{n+1}$ $\forall$ $n \in N$.

(iv) A sequence $\{a_n\}$ is said to be strictly monotonically decreasing if $a_n > a_{n+1}$ $\forall$ $n \in N$.

Eg. 1) $\{\frac{1}{n}\}$ is strictly monotonically decreasing.

2) $\{n^4\}$ is strictly monotonically increasing.

3) $\{(-1)^n\}$ is neither monotonically increasing nor monotonically decreasing.

★ Behaviour of Monotonic sequence

1) ~~Bounded~~ Monotonically increasing sequence $\{a_n\}$ converges iff it is bounded above.

(ii) Monotonically decreasing sequence $\{a_n\}$ converges iff it is bounded below.

iii) A monotonic sequence Converges iff it is bounded.

(iv) If monotonically increasing sequence $\{a_n\}$ unbounded. above then it is diverges to $+\infty$

v) Monotonically decreasing seq $\{a_n\}$ diverges to $-\infty$ iff ~~and~~ it is unbounded below.

$\Rightarrow$ Note

1) Monotonically increasing seq $\{a_n\}$ either Converges or diverges to $+\infty$

2) Monotonically decreasing Sequence $\{a_n\}$ either Converges or diverges to $-\infty$.

3) Monotonic Sequence can never be oscillatory.

4) A necessary and sufficient Condition for the Convergence of monotonic sequence is that it is bounded.

5) A necessary and sufficient Condition for the divergence of the monotonic sequence is that it is unbounded.

Cauchy's first theorem on limits.

If $a_n \to l$ then $x_n = \dfrac{a_1 + a_2 + \dots + a_n}{n} \to l$

Cauchy's second th^m on limits.

If $\{a_n\}$ is a ~~po~~ sequence of positive terms and let $\lim_{n\to\infty} \dfrac{a_{n+1}}{a_n}$ exist whether finite or infinite, then $\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \dfrac{a_{n+1}}{a_n}$

1. Prove that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is

(i) monotonically increasing. (ii) bounded (iii) Convergent

Ans (i) $a_n = \dfrac{2n-7}{3n+2}$

$$a_{n+1} = \frac{2(n+1)-7}{3(n+1)+2} = \frac{2n-5}{3n+5}$$

$$a_{n+1} - a_n = \frac{2n-7}{3n+2} - \frac{2n-5}{3n+5} = \frac{6n^2+4n-15n-10-6n^2}{(3n+2)(3n+5)}$$

$+21n-10n$
$+35$

$$= \frac{25}{(3n+5)(3n+2)} > 0 \quad \forall n \in N$$

$a_{n+1} > a_n \quad \forall n \in N$

$\{a_n\}$ is monotonically increasing.

(ii) $a_n = \dfrac{2n-7}{3n+2} > 0 \quad \forall \quad n \geq 4$

$a_1 = \dfrac{-5}{5} = -1 \qquad a_2 = \dfrac{-3}{8}, \; a_3 = \dfrac{-1}{11}$

$\therefore \; a_n \geq -1 \quad \forall n$

$\{a_n\}$ is bounded below

$$a_n = \frac{2n-7}{3n+2} = \frac{2}{3} - \frac{\frac{25}{3}}{3n+2} < \frac{2}{3} \quad \forall n$$

$\{a_n\}$ is bounded above.

$\therefore$ $\{a_n\}$ is bounded.

(iii)

Since $\{a_n\}$ is monotonically increasing and bounded (above) sequence.

$\therefore$ $\{a_n\}$ is convergent.

Eg. Prove that the sequence $\{a_n\}$ where ~~an=~~

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}$$

is convergent.

Sol Here $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$

$$a_{n+1} = \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} + \frac{1}{2n+1} + \frac{1}{2n+2}$$

$$a_{n+1} - a_n = \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}$$

$$= \frac{1}{2n+1} + \left[\frac{1}{2(n+1)} - \frac{1}{n+1}\right]$$

$$= \frac{1}{2n+1} - \frac{1}{2(n+1)} = \frac{1}{2n+1} - \frac{1}{2n+2}$$

$$= \frac{1}{(2n+1)(2n+2)} > 0$$

$a_{n+1} > a_n$ $\quad\therefore$ $\{a_n\}$ is monotonically increasing seq^n.

Also $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} < \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n}$

$= \frac{1}{n} + \frac{1}{n} + \dots$ to n terms $= \frac{n}{n} = 1 \quad \forall n \in N$

$a_n < 1$

$\Rightarrow \{a_n\}$ is bounded above.

$\Rightarrow \therefore \{a_n\}$ is Convergent.

☆ $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots$

$e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \dots$

Eg. Prove that the sequence $a_n = \left(1+\frac{1}{n}\right)^n$ is Convergent.

Sol. Here, $a_n = \left(1+\frac{1}{n}\right)^n = 1 + n\cdot\frac{1}{n} + \frac{n(n-1)}{\lfloor 2}\left(\frac{1}{n}\right)^2$

$+ \frac{n(n-1)(n-2)}{\lfloor 3}\left(\frac{1}{n}\right)^3 + \dots$ to $(n+1)$ terms.

$= 1 + 1 + \frac{1}{\lfloor 2}\left(1-\frac{1}{n}\right) + \frac{1}{\lfloor 3}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + \dots$

$\dots + \frac{1}{\lfloor n}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{n-1}{n}\right)$

$a_{n+1} = 1 + 1 + \frac{1}{\lfloor 2}\left(1-\frac{1}{n+1}\right) + \frac{1}{\lfloor 3}\left(1-\frac{1}{n+1}\right)\left(1-\frac{2}{n+1}\right) + \dots$

$+ \frac{1}{\lfloor n+1}\left(1-\frac{1}{n+1}\right)\left(1-\frac{2}{n+1}\right)\dots\left(1-\frac{n}{n+1}\right)$

Since $1-\frac{r}{n} < 1-\frac{r}{n+1} \quad \forall\, r, n \in N$

$\therefore$ each term in $a_n <$ corresponding terms in a_{n+1},

Also a_{n+1} contains one term more than the total number of terms of a_n,

$\therefore \quad a_n < a_{n+1} \quad \forall\, n \in N.$

$\therefore \quad \{a_n\}$ is an increasing seq.

also $a_n = 1 + \frac{1}{1} + \frac{1}{\underline{|2}}\left(1-\frac{1}{n}\right) + \cdots + \frac{1}{\underline{|n}}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{n-1}{n}\right)$

$\leq 1 + 1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \cdots + \frac{1}{\underline{|n}}$ or

$< 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \cdots + \frac{1}{\underline{|n}} + \cdots \infty$

$= e$

$a_n < e \quad \forall\, n \in N$

$\Rightarrow \{a_n\}$ is bounded above.

$\Rightarrow \{a_n\}$ is convergent.

Eg. Show ~~prove~~ that $a_n = (-1)^n$ does not converge.

Sol A sequence is convergent iff $\lim_{n\to\infty} a_n = l$ and l is a finite number.

But $\lim_{n\to\infty} a_n = \begin{cases} -1 & \text{for odd } n \\ 1 & \text{for even } n \end{cases}$

$\Rightarrow$ $\{a_n\}$ does not converge.

Eg. Show that $\{n^2+3n\}$ diverges to $+\infty$.

$\Rightarrow$ find $\lim_{n\to\infty} a_n$ *

if $\lim_{n\to\infty} a_n = \infty$ Seq^n is divergent and diverges to $+\infty$

$a_n = n^2+3n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(n^2+3n) = \infty$

Eg. Show that Seq^n $\{1+(-1)^n\}$ oscillates.

$\{0, 2, 0, 2, \text{---}\}$

$\lim_{n\to\infty} a_n = \begin{cases} 0 & \text{for odd } n \\ 2 & \text{for even } n \end{cases}$

Seq^n is oscillatory Seq^n.

note also check bounded but not Convergent. Seq^n is always oscillates.

Eg. Show that $a_n = \dfrac{n^2+3n+5}{2n^2+5n+7}$ Converges to $\dfrac{1}{2}$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\cancel{n^2}(1+3/n+5/n^2)}{\cancel{n^2}(2+5/n+7/n^2)} = \frac{1}{2}$$

Q Show that Seq^n $a_n = (2-n^2)$ diverges to $-\infty$

H.W

Eg Prove that $\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) = 0$

$a_n = \frac{1}{n}$

then $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

by Cauchy's first Th^m.

$$\lim_{n\to\infty} \frac{a_1+a_2+\cdots+a_n}{n} = \lim_{n\to\infty} \frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{n} = \lim_{n\to\infty} a_n$$

$= 0.$

Eg. $\lim_{n\to\infty} \left[\left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n} = e$

by Cauchy's Second Th^m.

$a_n = \left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n$

$a_{n+1} = \left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\left(\frac{n+2}{n+1}\right)^{n+1}$

$\frac{a_{n+1}}{a_n} = \left(\frac{n+2}{n+1}\right)^{n+1} = \left(1+\frac{1}{n+1}\right)^{n+1}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\left(1+\frac{1}{n+1}\right)^{n+1} = e$$

Infinite Series.

Let $\{a_n\}$ be a real sequence.

The expression $a_1 + a_2 + \ldots + a_n + \ldots$ is called infinite series and is denoted by $\sum_{n=1}^{\infty} a_n$ or by $\sum a_n$ and a_n is called n^{th} term of the series.

Partial Sums.

we define

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_n = a_1 + a_2 + \ldots + a_n = \sum_{1}^{n} a_n$

These S's are called Partial Sums of the series $\sum a_n$.

The n^{th} Partial sum is ~~def~~ denoted as S_n or s_n or σ_n

Behaviour of an Infinite Series.

1) Convergent Series:- The Series $\sum a_n$ is said to be Convergent if the sequence of its partial sums $\{S_n\}$ Converges to S and S is called the sum of the convergent infinite series $\sum_{1}^{\infty} a_n$. we write $\sum_{1}^{\infty} a_n = S$.

(ii) Divergent Series: The Series Σa_n is said to be divergent to ∞ (or $-\infty$) if the sequence $\{S_n\}$ diverges to ∞ (or $-\infty$). We write $\sum_1^{\infty} a_n = \infty$ or $\sum_1^{\infty} a_n = -\infty$

$\downarrow$ diverges to $+\infty$ $\qquad$ $\downarrow$ diverges to $-\infty$

(iii) Oscillatory Series: The Series Σa_n is said to oscillate finitely or infinitely if the sequence $\{S_n\}$ oscillates finitely or infinitely.

(iv) Asolutely Convergent Series:- The series Σa_n is said to be ~~Convergent~~ absolutely if the series $\sum_1^{\infty} |a_n|$ i.e. the series $|a_1| + |a_2| + |a_3| + \cdots$ is Convergent.

(v) Conditionally Convergent Series. The Series Σa_n is said to be Conditionally convergent if Σa_n is Convergent but $\Sigma |a_n|$ is not Convergent.

(vi) Non-Convergent Series:- Series which diverge or Oscillate are said to be non-convergent.

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5:, Nature of Geometric Series.

Show that the geometric series $\sum_{1}^{\infty} r^{n-1} = 1 + r + r^2 + \dots$

(i) Converges to $\frac{1}{1-r}$ for $|r|<1$ (ii) diverges to $+\infty$ if $r \geq 1$

(iii) Oscillates finitely between 0 and 1 if $r=-1$ (iv) Oscillate infinitely if $r<-1$

(v) Converges absolutely for $|r|<1$.

Sol^n:→ The given series $1 + r + r^2 + r^3 + \dots$

Partial Sums of the series.

$S_1 = 1$; $S_2 = 1+r$; $S_3 = 1+r+r^2$; $S_n = 1+r+r^2+\dots+r^{n-1}$

$= \frac{1(1-r^n)}{1-r}$ for $|r|<1$

$= \frac{1(r^n-1)}{r-1}$ $r>1$

(1) for $|r|<1$

$\therefore S_n = \frac{1(1-r^n)}{1-r} = \frac{1-r^n}{1-r} = \frac{1}{1-r} - \frac{r^n}{1-r}$

$\lim_{n\to\infty} S_n = \frac{1}{1-r}$ $\left[\because r^n \to 0 \text{ as } n\to\infty \quad \because |r|<1\right]$

$\therefore \{S_n\}$ is Convergent Sequence.

$\Rightarrow \sum a_n$ Converges to $\frac{1}{1-r}$ for $|r|<1$

(ii) for $r=1$

Series is $\sum a_n = 1+1+1+\dots \infty$ times.

$\therefore \sum a_n$ diverges to $+\infty$

for $r>1$

$\therefore S_n = \frac{r^n-1}{r-1} \Rightarrow \lim_{n\to\infty} S_n = \infty$ $\left[\text{as } r^n \to \infty \text{ as } n\to\infty \text{ for } r>1\right.$

$\therefore \sum a_n$ is diverges to $+\infty$ for $r \geq 1$

(iii) for $r = -1$

$$S_n = 1-1+1-1 \dots n \text{ terms} = \begin{cases} 0 & \text{if } n \text{ is } \cancel{\text{odd}}\ \text{even} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$$

$\therefore \{S_n\}$ oscillates finitely between 0 and 1 for $r=-1$

$\Rightarrow \Sigma a_n$ oscillates finitely between 0 and 1 for $r=-1$

(iv) for $r < -1$

$$S_n = \frac{r^n - 1}{r-1} \qquad \left[\because r^n \to \begin{cases} \infty & \text{for even} \\ -\infty & \text{for odd} \end{cases} \quad r<-1 \right.$$

$$\lim_{n\to\infty} S_n = \begin{cases} \infty & \text{for even } n \\ -\infty & \text{for odd } n \end{cases}$$

$\{S_n\}$ oscillate infinitely for $r < -1$

$\Rightarrow \Sigma a_n$ oscillates infinitely.

(v) for $|r| < 1$

$$\Sigma |a_n| = 1 + |r| + |r|^2 + |r|^3 + \dots$$

$$S_n = \frac{1-|r|^n}{1-|r|} = \frac{1}{1-|r|} - \frac{|r|^n}{1-|r|}$$

$$\lim_{n\to\infty} S_n = \frac{1}{1-|r|} \qquad \left[\because |r|^n \to 0 \text{ as } n\to\infty \text{ for } |r|<1 \right.$$

$\Sigma |a_n|$ Convergent Series.

$\Rightarrow \Sigma a_n$ Converges absolutely.

H.W Prove above results for $\sum_1^\infty ar^{n-1} = a + ar + ar^2 + \dots$

if $\sum_1^\infty a_n$ is Convergent, then prove that

$$\lim_{n\to\infty} a_n = 0$$

Proof :→ Let $S_n = a_1 + a_2 + \dots + a_n$

$\because \sum a_n$ is Convergent to S

$\therefore \{S_n\}$ is convergent to S

$\Rightarrow \{S_{n-1}\}$ is Convergent to S [$\because$ every subsequence of convergent seq is cgt

Now
$a_n = S_n - S_{n-1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(S_n - S_{n-1}) = \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1}$$

$$= S - S = 0$$

$$\lim_{n\to\infty} a_n = 0$$

★ Also if the Sequence $\{a_n\}$ does not converge to zero, then $\sum a_n$ does not Converge.

★ The necessary Condition for Convergence of $\sum a_n$ is $a_n \to 0$.

Eg. 1 (H.W) Show that the geometric series $\sum_0^\infty r^n$, where r is any real such that $|r|<1$ is Convergent [PTU 2017]

Eg. 2. Show that $1 + \frac{1}{2} + \frac{1}{2^2} + \dots$ Converges to 2

Eg 3. Show that $\sum_{1}^{\infty} \frac{1}{n(n+1)} = 1$

Sol^n the given series $\sum a_n$ where $a_n = \frac{1}{n(n+1)}$

$\Rightarrow a_n = \frac{1}{n(0+1)} + \frac{1}{(-1)(n+1)} \Rightarrow a_n = \frac{1}{(n)} - \frac{1}{(n+1)}$

$a_1 = \frac{1}{1} - \frac{1}{2}$; $a_2 = \frac{1}{2} - \frac{1}{3}$; $a_3 \ \frac{1}{3} - \frac{1}{n}$

$a_n = \frac{1}{n} - \frac{1}{n+1}$

$S_n = 1 - \frac{1}{n+1}$

$\lim_{n\to\infty} S_n = 1 - 0 = 1$ $\therefore \sum a_n$ is convergent

$\Rightarrow \sum a_n = 1$

Eg. Show that the series $1^2 + 2^2 + 3^2 + \dots + n^2 + \dots$ diverges to $+\infty$

Sol: $\rightarrow$ given Series is $1^2 + 2^2 + \dots + n^2 + \dots$

$S_n = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty} S_n = \infty \Rightarrow \sum_{1}^{\infty} a_n = \infty$

$\sum a_n$ diverges to $+\infty$.

Eg Give an example of an oscillating series.

$\sum a_n$ where $a_n = (-1)^{n-1} \Rightarrow S_n = \begin{cases} 0 & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$

$\{S_n\}$ oscillates $\Rightarrow \sum a_n$ oscillate.

ests for Positive term Series.

1) Comparison test for +ve term series.

k positive constant depending on a_n, b_n.

(i) $a_n \le k b_n$	If Σb_n convergent then Σa_n is also convergent.
(ii) $a_n \ge k b_n$	if Σb_n diverges then Σa_n is also diverges.
(iii) $\lim_{n\to\infty} \frac{a_n}{b_n} = l$	(i) l (finite and non-zero.) $\Rightarrow \Sigma a_n, \Sigma b_n$ have same nature. (ii) $l = 0$ if Σb_n converges $\Rightarrow \Sigma a_n$ converges. (iii) $l = \infty$ if Σb_n diverges $\Rightarrow \Sigma a_n$ diverges.

for all $n \ge m$ particular stage

Sr.No	Name of test	formula	behaviour.
2)	Geometric series	$1+r+r^2+\dots+r^{n-1}+\dots = \sum_{1}^{\infty} r^{n-1}$	(i) Convergent if $\lvert r\rvert<1$ (ii) diverges to $+\infty$ if $r \ge 1$ (iii) Oscillates if $r \le -1$
3)	Cauchy's Root test.	$\lim_{n\to\infty} (a_n)^{1/n} = l$	i) Convergent if $l<1$ ii) divergent if $l>1$ iii) test fails if $l=1$
4)	p-test	$\Sigma \frac{1}{n^p}$	i) Convergent if $p>1$ ii) divergent if $p \le 1$
5)	Cauchy's integral test	$\int_1^{\infty} f(x)\,dx = l$	i) If l finite then convergent ii) If l infinite then divergent.

Sr. No	Name of test	Formula	behaviour
6.	D-Alembert Ratio test.	$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = l$	(i) Convergent if $l>1$ (ii) divergent if $l<1$ (iii) test fails if $l=1$
7.	Raabe's test (used when ratio test fails)	$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = l$	i) Convergent if $l>1$ ii) divergent if $l<1$ iii) test fails if $l=1$
8.	Logarithmic test (used when ratio test fails and limit involves exponential e)	$\lim_{n\to\infty} n \log\left(\frac{a_n}{a_{n+1}}\right) = l$	i) Convergent if $l>1$ ii) divergent if $l<1$ iii) test fails if $l=1$
9.	Gauss's test	Expand $\frac{a_n}{a_{n+1}} = 1 + \frac{\lambda}{n} + O\left(\frac{1}{n^2}\right)$	i) Convergent if $\lambda>1$ ii) divergent if $\lambda \leq 1$